$$\begin{aligned} Term_5 =& P^{\mathrm{Ie}}_{s,p} P^{\mathrm{II}h}_{n,r} \pi A_n \int_0^b \left[n J'_s\left(k^{\mathrm{Ie}}_{c_{s,p}}\rho\right) J_n\left(k^{\mathrm{II}h}_{c_r}\rho\right) + s J_s\left(k^{\mathrm{Ie}}_{c_r}\rho\right) \right. \\ & \left. \cdot J'_n\left(k^{\mathrm{II}h}_{c}\rho\right)\right] d\rho, \qquad \text{if } n = s \\ =& 0, \qquad \text{if } n \neq s \end{aligned} \tag{21}$$

$$\begin{aligned} Term_6 =& P^{\mathrm{Ie}}_{s,p} P^{\mathrm{II}h}_{n,r} C_n \int_b^a -l J'_s\left(k^{\mathrm{Ie}}_{c_{s,p}}\rho\right) \left[J_l\left(k^{\mathrm{II}h}_{c_r}\rho\right) N'_l\left(k^{\mathrm{II}h}_{c_r} a\right) \right. \\ & \left. - J'_l\left(k^{\mathrm{II}h}_{c_r} a\right) N_l\left(k^{\mathrm{II}h}_{c_r}\rho\right)\right] \\ & \cdot d\rho \times 4 \int_\theta^{\frac{\pi}{2}} \cos s\phi \sin l(\phi - \theta)\, d\phi + P^{\mathrm{Ie}}_{s,p} P^{\mathrm{II}h}_{n,r} C_n \\ & \cdot \int_b^a s J_s\left(k^{\mathrm{Ie}}_{c_{s,p}}\rho\right) \left[J'_l\left(k^{\mathrm{II}h}_{c_r}\rho\right) N'_l\left(k^{\mathrm{II}h}_{c_r} a\right) - J'_l\left(k^{\mathrm{II}h}_{c_r} a\right) \right. \\ & \left. \cdot N'_l\left(k^{\mathrm{II}h}_{c_r}\rho\right)\right] \\ & \cdot d\rho \times 4 \int_\theta^{\frac{\pi}{2}} \sin s\phi \cos l(\phi - \theta)\, d\phi \end{aligned} \tag{22}$$

where $l = (n-1)\pi/\phi_o$.

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Simple Determination of All Capacitances for a Set of Parallel Microstrip Lines

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***Abstract*— A fast and moderately accurate method to describe the complicated dependence on design and process parameters of coupling capacitances between a set of parallel lines is presented in this paper. It involves only one circuit-dependent parameter at a time. This is accomplished by calculating the capacitance coefficient matrix through inversion of a potential coefficient matrix with much simpler dependence on geometry. Self elements are approximately independent of the presence of other lines, and mutual elements do not depend on linewidths or interfering lines as long as the ground is sufficiently far away. The potential coefficients are derived by inverting one- or two-line capacitance matrices that are either theoretically calculated or determined by measurements on integrated circuit (IC) test structures. Look-up tables for a specific IC process can then be constructed with only linewidth as the parameter for self potential elements and distance between line centers as parameter for mutual potential elements. General algorithms have been derived for microstrip on one or two layers of dielectric.**

***Index Terms*— Coupled lines, design automation software, wiring models.**

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I. Introduction

As digital and mixed analog/digital integrated circuits (IC's) move higher and higher in speed and increase in size, complexity, and packing density, the need rises to include moderately accurate models of line-to-line and line-to-ground capacitances for the estimation of performance degradation and crosstalk. When line length is not negligible compared to wavelength, inductances must also be taken into account. Much work has been devoted to alleviate the burden on computers due to the inclusion of parasitic elements in the circuit simulators [1]. Separate programs for evaluation of parallel lines are available [2]. One recent work [3] describes the generation of analytical models for interconnect capacitances in the form of polynomials in the design parameters given the values of process parameters. Contrary to the method described in this paper, every addition of another line necessitates the determination of a new set of curves with a rapid increase in the number of parameters (N lines give $2N-1$ parameters).

Our method is based on the observation that parallel lines propagating TEM waves are described by an inductance matrix with—to a first approximation—self elements depending only on the perimeter of the line cross section, and mutual elements depending only on the distance between line centers, without being influenced by the addition of new lines, whether shielding or not. The capacitance matrix is derived by inversion of the complete inductance matrix for the set of N lines. With inhomogeneous dielectric outside the conductors, wave propagation is quasi-TEM, and this simple relation breaks down. It is found, though, that the elements of the potential coefficient matrix (the inverse of the capacitance coefficient matrix) show the same simple dependence on design parameters as do the inductance elements.

II. General TEM Relations

For a set of N parallel conductors over a ground surface and propagating TEM waves, one can define inductances (self and mutual) and capacitances between conductors. The inductance matrix is $\mu_0[L_{ij}]$ H/m and the capacitance coefficient matrix is $\varepsilon_0[c_{ij}]$ F/m, both of rank $N \times N$. If the whole space outside the conductors is filled with a dielectric with permittivity ε, the following well-known relation is true:

$$[c_{ij}] = \varepsilon[L_{ij}]^{-1}. \tag{1}$$

The physical capacitances per unit line length (normalized to ε_0) between conductor i and ground C_{i0} and between conductors i and j, C_{ij} are connected to the capacitance coefficients

$$\begin{aligned} C_{i0} &= \Sigma_j c_{ij} \\ C_{ij} &= -c_{ij}, \qquad \text{with } i \neq j. \end{aligned} \tag{1a}$$

In a strict sense, (1) is true only when the currents are confined to the surface of the conductors (infinite conductivity)—or when the distance between conductors and to the ground surface is much larger than their transverse dimensions.

III. Self-Inductance

For a single conductor over a ground plane, the inductance per unit length depends on one dimensionless variable: the transverse perimeter length of the conductor divided by the distance between some point in the conductor and the ground plane. The addition of more conductors is found to disturb this relation only marginally and, therefore, one may assume that this dimensionless variable and the functional dependence on it holds approximately true for all L_{ii} in the presence of the other conductors.

For a *circular wire* $(i = 1)$ over ground, we have [4, formula set L-2]

$$L_{11} = \frac{1}{2\pi}\left\{\frac{1}{4} + \ln\left(\frac{4\pi}{P_1}\right)\right\} \tag{2}$$

where the first term is the internal contribution to the self-inductance, and the normalized perimeter of the wire is

$$P_1 = 2\pi\frac{r_1}{h_1}$$

where r_1 is the wire radius and h_1 is the distance between the wire center line and ground plane.

Formula (2) is valid for an even current distribution within the wire, i.e., at dc. With extreme skin effect (high conductivity and/or frequency), we have instead [5, formula (19, 45)]

$$L_{11} = \frac{1}{2\pi}\ln\left\{\frac{2\pi}{P_1} + \sqrt{\left(\frac{2\pi}{P_1}\right)^2 - 1}\right\}. \tag{2a}$$

Formula (2) gives a value exceeding (2a) with at least the wire's internal contribution. The choice between (2) and (2a) depends on frequency and wire resistivity. With an even current distribution within the wires it can be shown [5, formula (19), 54–55] that (2) is exactly valid for each one in a set of parallel wires. A suitable compromise for the self-inductances of a set of wires is, therefore,

$$L_{ii} \cong \frac{1}{2\pi}\ln\left(\frac{2h_i}{r_i}\right). \tag{2b}$$

For a *rectangular strip* $(i = 1)$ over ground, we have [4, formula set L-5]

$$L_{11} \cong \frac{1}{2\pi}\ln\left(1 + \frac{4\pi}{P_1}\right) \tag{3}$$

where the normalized perimeter of the strip is

$$P_1 = 2\frac{W_1 + T_1}{H_1}$$

where W_1 is the strip width, T_1 is the strip thickness, and H_1 is the free distance between strip and ground plane.

We obtain an approximate expression for the self-inductances of a set of rectangular strips as follows:

$$L_{ii} \cong \frac{1}{2\pi}\ln\left\{1 + 2\pi\frac{H_i}{W_i + T_i}\right\}. \tag{3a}$$

The accuracy of (3a) is found to be well within 10% when checked over a wide parameter range. About half of the total variation can be ascribed to presence of other conductors.

It is clear that (3) and (3a), like (2a), describe a situation with only surface currents, as the limiting value is zero for a conductor touching ground. An approximate way of introducing a finite skin depth is to displace the conductor surfaces inwards an amount equal to half the skin depth δ_s [6]. This would imply that $r_1 \Rightarrow r_1 - \delta_s/2$ and $h_1 \Rightarrow h_1 + \delta_s/2$ in (2a). When $\delta_s > 0.4r_1$, (2) will take over. In (3), W_1 and T_1 are decreased by δ_s and H_1 is increased by δ_s as long as the changes in W_1 and T_1 are less than about 20%.

IV. Mutual Inductance

The mutual inductance between two parallel conductors over a ground plane is characterized by one dimensionless variable: the distance between center lines through the conductors divided by the geometric mean of the heights of the respective conductors over the ground plane as defined from some point in the conductor. Dielectrics in the space between conductors do not influence inductance. Adding more conductors disturbs the value of the mutual inductance between the original pair of conductors very little; only when the distance is much longer than the height over ground and the conductors in between are filling the space to a high degree can a nonnegligible influence be observed.

The mutual inductance between two parallel conductors over a ground plane is given in [4, formula set L-3] for *circular wires* and in [4, formula set L-7] as an approximation for *rectangular strips*:

$$L_{12} = \frac{1}{4\pi}\ln\left\{1 + \left(\frac{2}{D_{12}}\right)^2\right\} \tag{4}$$

where the normalized distance between the conductors is

$$\text{for circular wires:} \quad D_{12} = \frac{d_{12}}{\sqrt{h_1 h_2}},$$

$$\text{for rectangular strips:} \quad D_{12} = \frac{d_{12}}{\sqrt{\left(H_1 + \frac{T_1}{4}\right)\left(H_2 + \frac{T_2}{4}\right)}}$$

where d_{12} is the distance between the conductor center lines.

We then have an approximate expression for the mutual inductance of a set of parallel conductors as follows:

$$L_{ij} \cong \frac{1}{4\pi}\ln\left\{1 + \left(\frac{2}{D_{ij}}\right)^2\right\}, \qquad \text{with } i \neq j. \tag{4a}$$

This formula is exact for circular wires with even current distribution [5, formula (19, 54–55)]. For rectangular strips, the accuracy of (4a) is found to be within 30% (within 10% for $D_{ij} < 1.2$). About half of the total variation can be ascribed to presence of other conductors.

V. Capacitance

If the space outside the conductors is homogeneously filled with a dielectric, relation (1) may be used and the capacitances determined. For circular bonding wires in air, this is the case: $(\varepsilon = 1)$. Observe that formula (2a) with surface currents must be used to derive capacitance values.

In the case of microstrip lines with a substrate below rectangular strips and air above, one cannot proceed that easily. The capacitance coefficients may be determined for a large number of cases with the help of numerical methods like the method-of-lines (MoL) [7]. We have done that and inverted the capacitance coefficient matrix to obtain a matrix analogous to an inductance matrix. It is found that the elements in this potential coefficient matrix behave similarly to the inductances, i.e., self elements depend only on the P-parameter plus the substrate permittivity and the mutual elements depend only on the D-parameter plus the substrate permittivity. As is the case for the inductances, this is only approximately true, but it enables a very fast and simple method of calculation. The elements of this potential coefficient matrix are called Λ_{ij}, and a procedure of fitting to the numerical calculations—choosing appropriate algorithms with the right asymptotic behavior for air as substrate—yields the following formulas:

$$\Lambda_{ii} = \frac{1}{2\pi}\ln\left(1 + \frac{4\pi}{\prod_i}\right) \tag{5}$$

where

$$\prod_i = \mathrm{Er} \cdot P_i + \alpha \cdot P_i^{\beta/(1+1.6 \cdot P_i)}$$

with

$$P_i = \frac{2(W_i + T_i)}{H_i + \frac{T_i}{2}\left(1 - \frac{1}{\mathrm{Er}}\right)}$$

and

$$\alpha = 1.07(Er - 1)^{1.15}$$
$$\beta = \ln\left(1.15 + \frac{1.17}{\mathrm{Er}}\right).$$

Er (in the range 1–20) is the substrate permittivity

$$\Lambda_{ij} = \frac{1}{4\pi} \ln\left\{1 + \left(\frac{2}{\Delta_{ij}}\right)^2\right\}, \qquad \text{with } i \neq j \tag{6}$$

where

$$\Delta_{ij} = \gamma \cdot D_{ij}^{1.5} + \delta \cdot D_{ij}^{\eta}$$

with

$$D_{ij} = \frac{d_{ij}}{\sqrt{\left(H_i + \frac{T_i}{4}\right)\left(H_j + \frac{T_j}{4}\right)}}$$

and

$$\gamma = 3.6 \tanh\{0.09 \cdot (\mathrm{Er} - 1)\}$$
$$\delta = 1 + 0.36 \cdot (\mathrm{Er} - 1)^{0.65}$$
$$\eta = \mathrm{Er}^{-0.66}.$$

The accuracy of (5) is similar to that of (3a), and the accuracy of (6) is similar to that of (4a) with the restriction that $D_{ij} < 3$. For larger D_{ij}-values, (6) gets more and more inaccurate. Now, the capacitance coefficient matrix is given simply by

$$[c_{ij}] = [\Lambda_{ij}]^{-1} \tag{7}$$

with capacitances as in (1a). C_{i0} turns out to be accurate within 10%, while C_{ij} may be 40% in error for $D_{ij} < 1.5$ and still more for larger D_{ij}-values. In the homogeneous case with $\mathrm{Er} = 1$, C_{ij} stays within 30% deviation from the true value.

To show the usefulness (and limitation) of the method described above, a comparison between results in [8] and our algorithms for a four-line system and a five-line system (as illustrated in Fig. 1) is given in Table I. In [8], the MoL has been used.

Errors in the approximate values of L and C from Table I are roughly four times higher in the five-line than in the four-line system. This is connected to the fact that the former system is twice as wide as it is high, while the width of the latter is only 60% of its height. Probably most relevant for estimation of parasitic influence is the ratio between the maximum absolute error in partial capacitance and the largest among the partial capacitances [3]. This error is 3% (four-line) and 13% (five-line).

VI. Capacitance With Layered Substrate

Microstrip lines very often use a layered substrate, especially in digital IC and monolithic microwave integrated circuit (MMIC) applications. The formulas in Section V may be used if the substrate permittivity Er is substituted by equivalent values that will be different for (5) and (6).

With a two-layer substrate consisting of a grounded layer with permittivity Eg and a coating below the strip with permittivity Ec and thickness Tc, the following expressions for the equivalent value of substrate permittivity give a good fit to numerical calculations and have the appropriate asymptotic behavior for $\mathrm{Ec} = \mathrm{Eg} = \mathrm{Er}$ and for $\mathrm{Tc} = 0$ or $\mathrm{Tc} = H$:

$$\mathrm{Er}_{\mathrm{eq}} = \{k \cdot \mathrm{Ec}^s + (1 - k) \cdot \mathrm{Eg}^s\}^{1/s} \tag{8}$$

with

$$s = -\left(\frac{\mathrm{Tc}_i}{H_i}\right)^{0.4}$$

and

$$k = \left(\frac{\ln\left(1 + \frac{4\mathrm{Tc}_i}{W_i}\right)}{\ln\left(1 + \frac{4H_i}{W_i}\right)}\right)^{0.8}, \qquad \text{for } \Lambda_{ii},$$

and with

$$s = -\left(\frac{\mathrm{Tc}_i \cdot \mathrm{Tc}_j}{H_i \cdot H_j}\right)^{0.2}$$

and

$$k = \frac{\ln\left(1 + 5\frac{\mathrm{Tc}_i \cdot \mathrm{Tc}_j}{d_{ij}^2}\right)}{\ln\left(1 + 5\frac{H_i \cdot H_j}{d_{ij}^2}\right)}, \qquad \text{for } \Lambda_{ij} \text{ with } i \neq j.$$

A comparison between results with (5)–(8) and the MoL is given in Table II for two coupled microstrips.

The largest error in Λ-value in Table II is 13% and in C-value 26%. The tendency is usually that the error increases on inversion of the Λ-matrix.

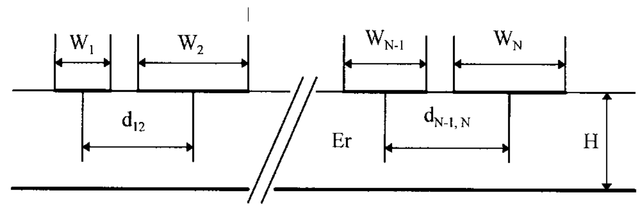

Fig. 1. Cross section of an N-conductor microstrip system.

TABLE I
INDUCTANCE AND CAPACITANCE MATRIX ELEMENTS. COMPARISON BETWEEN THE PRESENT APPROXIMATION AND [8, TH. 4, 5]. FOUR-LINE SYSTEM: $\mathrm{Er} = 9.8$, $W/H = 0.102$, $d/H = 0.173$, $W_1 = W_2 = W_3 = W_4 = W$, $d_{12} = d_{23} = d_{34} = d$. FIVE-LINE SYSTEM: $\mathrm{Er} = 2.5$, $H = 0.8$ mm, $W_1 = W_3 = W_5 = 0.16$ mm, $W_2 = W_4 = 0.47$ mm, $d_{12} = d_{23} = d_{34} = d_{45} = 0.385$ mm

SOURCE		Proposed Formulas (2) to (4) L_{ij}	(5) to (7) C_{ij}	Reference [8, Tables 4 & 5] L_{ij}	C_{ij}
4-line system:					
L_{11}	C_{10}	0.6584	4.963	0.6777	4.7965
L_{12}	C_{12}	0.3901	5.851	0.3866	5.8084
L_{13}	C_{13}	0.2816	0.961	0.2829	1.1476
L_{14}	C_{14}	0.2199	0.468	0.2227	0.6184
L_{22}	C_{20}	0.6584	2.781	0.6709	2.8953
L_{23}	C_{23}	0.3901	5.429	0.3840	5.3306
5-line system:					
L_{11}	C_{10}	0.5536	1.878	0.5404	1.8009
L_{12}	C_{12}	0.2312	2.250	0.2451	2.5889
L_{13}	C_{13}	0.1330	0.0376	0.1578	0.0880
L_{14}	C_{14}	0.0852	0.0917	0.1039	0.0791
L_{15}	C_{15}	0.0583	0.0361	0.0766	0.0229
L_{22}	C_{20}	0.3914	2.354	0.3968	2.2502
L_{23}	C_{23}	0.2312	2.062	0.2332	2.3511
L_{24}	C_{24}	0.1330	0.553	0.1449	0.5720
L_{33}	C_{30}	0.5536	0.851	0.5075	0.8524

TABLE II
CAPACITANCE AND POTENTIAL COEFFICIENT MATRIX ELEMENTS FOR TWO COUPLED MICROSTRIPS OF EQUAL WIDTH AND ZERO THICKNESS ON A 105-MM-THICK SUBSTRATE (5-μm SiN, $Ec = 4.1$, ON TOP OF 100-μm InP, $Eg = 12.5$). FORMULAS (5)–(8) COMPARED TO MOL [7]

	W [μm]	d [μm]	Λ_{11}	Λ_{12}	C_{10}	C_{12}
MoL	3.75	4.5	0.1965	0.1174	3.186	4.726
Eq. (5)-(8)			0.2013	0.1080	3.233	3.741
MoL	3.75	10.5	0.2029	0.0700	3.666	1.930
Eq. (5)-(8)			0.2013	0.0708	3.674	1.994
MoL	12.5	25	0.1334	0.0430	5.668	2.695
Eq. (5)-(8)			0.1320	0.0433	5.703	2.788
MoL	12.5	40	0.1338	0.0307	6.082	1.808
Eq. (5)-(8)			0.1320	0.0308	6.143	1.867
MoL	12.5	80	0.1339	0.0156	6.693	0.880
Eq. (5)-(8)			0.1320	0.0146	6.820	0.849
MoL	37.5	138	0.0835	0.0069	11.07	0.990
Eq. (5)-(8)			0.0838	0.0061	11.13	0.872

VII. CONCLUSION AND PHYSICAL IMPLEMENTATION

We have shown that the proposed method using the simplified parameter dependence of the inverted capacitance coefficient matrix to generate capacitance values is applicable to very general configurations of parallel lines. Shielding effects from inserted lines and proximity effects on ground capacitance are taken care of automatically. The assumptions do not necessarily hold for overlapping microstrip lines, i.e., when the ground plane is shielded.

For an IC process with a semiisolating substrate, both accurate numerical and experimental determination of parametrized look-up tables is a straightforward task. Test structures with two coupled lines of standard width (W_i) and covering a range of distances (d_{ij}) could be experimentally characterized in a network analyzer at frequencies low enough to ensure pure capacitive behavior, but high enough to yield accurate results (usually 50–500 MHz).

When the bulk substrate is resistive, as with Si, things get more complicated. Inductance is calculated with line distance to ground equal to the full chip thickness as long as skin depth in the bulk is much greater than bulk thickness, i.e., for frequencies below a value proportional to bulk resistivity divided by bulk thickness squared (about 6 GHz for 1-mm-thick 10-Ω·cm Si). For most practical cases, the frequency is low enough. Capacitance consists of a series connection of an oxide layer and a lossy bulk. Below the frequency, where $\omega\varepsilon\varepsilon_0$ equals σ, the bulk capacitor tends to be short circuited. Above that frequency, the displacement current dominates the space down to ground; in between there is a lossy region. For 10-Ω·cm Si, this dividing frequency is 15 GHz, but for 1000 Ω·cm it is 150 MHz. Thus, a 10-Ω·cm Si process may safely be characterized at 50–500 MHz and yields values typical for the oxide layer. A 1000-Ω·cm Si process intended for applications at gigahertz frequencies will work with lower capacitance to ground corresponding to the full substrate thickness, and characterization should be made at 500–1000 MHz.

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Comparison Between Theoretical and Measured Microstrip Gap Parameters Involving Anisotropic Substrates

Jesús Martel, Francisco Medina, Rafael R. Boix, and Manuel Horno

***Abstract*— In this paper, experimental results are presented for microstrip symmetrical-gap discontinuities. The experimental technique is based on the measurement of the resonant frequencies of several gap-coupled rectangular microstrip resonators. In particular, gap discontinuities on anisotropic dielectric and two-layer composite substrates have been investigated. Reasonably good agreement has been found in most cases between theoretical data [obtained by means of the excess charge technique in the spectral domain (EC-SDA)] and experimental data, even though the theoretical results have been obtained by using a quasi-static approach.**

***Index Terms*—Anisotropic media, microstrip discontinuities, microwave measurements.**

I. INTRODUCTION

The correct characterization of the microstrip gap effect is essential for accurately predicting the frequency response of filters based on end-to-end coupled rectangular resonators [1]. In a former paper, the authors used the excess charge technique in the spectral domain (EC-SDA) in order to obtain numerical results for the equivalent circuit parameters of some microstrip gap discontinuities [2]. This technique is electrostatic in nature, but it has been reported that the equivalent-circuit parameters of microstrip gap discontinuities show a very slight variation with frequency [3]. In this paper, the quasi-static results obtained with the technique presented in [2] are compared with measured results. In particular, emphasis is placed on testing gap discontinuities printed on anisotropic substrates and composite two-layer substrates.

In this paper, a resonance technique is used for measuring the equivalent lengths of symmetric microstrip gaps. This type of experimental setup has been traditionally employed in the literature for characterizing microstrip discontinuities [3]–[8]. The technique is based on the measurement of the resonant frequencies of resonant

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